A Hausdorff topology on the future causal boundary: yet another rapprochement with conformal boundaries

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ABSTRACT

A natural Hausdorff topology τ_c is defined on the future causal completion $\hat{M}$ of a spacetime M with the following desirable features: the chronological future and pasts of points in $\hat{M}$ are open, future-directed sequences converge, M is a open dense set in $\hat{M}$ and the inclusion $i : M \to \hat{M}$ is open and continuous. In particular, the induced topology in M coincides with the manifold topology. The topology τ_c is shown to be strictly finer than the future chronological topology introduced in [1] and extensively discussed and championed in [2]. Inspired by recent work of O. Müller [3], this topology is compared to the future conformal *boundary of M induced by a conformal extension $M \hookrightarrow \tilde{M}$ such that (i) $\tilde{M}$ is a globally hyperbolic spacetime, (ii) $M \subset I^-(K, \tilde{M})$ for some compact set $K \subset \tilde{M}$ and (iii) M is causally convex in $\tilde{M}$. Of course, the standard conformal extensions of Minkowski, Schwarzschild and Robertson-Walker spacetimes are of this sort. Although τ_c is* a priori *distinct from the (also Hausdorff) topology defined on the future causal completion in [3], it reproduces results obtained therein for the future conformal boundary $\partial^+ M := I^+(M, \tilde{M}) \cap \partial M$, namely (a) that $\partial^+ M$ is a C^0 achronal hypersurface in $\tilde{M}$ homeomorphic to a Cauchy hypersurface in M, and (b) $M \cup \partial^+ M$ endowed with the topology induced by its inclusion in $\tilde{M}$ is homeomorphic to $\hat{M}$ with the topology τ_c. The homeomorphism maps the conformal boundary onto the future causal boundary, so that both boundaries coincide and are homeomorphic to Cauchy hypersurfaces in M.*

References

[1] J.L. Flores, *The Causal Boundary of spacetimes revisited*, Commun. Math. Phys. 276 (2007), 611-643.

[2] J.L. Flores, J. Herrera and M. Sánchez, *On the final definition of the causal boundary and its relation with the conformal boundary*, Adv. Theor. Math. Phys. Volume 15 (2011), 991-1058. arXiv:1001.3270

[3] O. Müller, *Which spacetimes admit conformal compactifications?*, to appear in Adv. Theor. Math. Physics (2017), arXiv:1409.8136.