

# Geodesic completeness in Lorentzian warped products

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## ABSTRACT

*Most singularity theorems in general relativity are based on the existence of trapped submanifolds with co-dimension two. In [1] the authors show that this can be generalized: they prove singularity theorems by using trapped submanifolds with arbitrary co-dimension. In particular, they provide a key, sufficient, trapping condition (\*) for the existence of focal points.*

*Let  $M$  be a Lorentzian manifold of dimension 4,  $\mathcal{Y}$  a compact Riemannian manifold of dimension 6, and consider the warped product  $\mathcal{M} = M \times_f \mathcal{Y}$ . We make a systematic study of the spacelike submanifolds of  $\mathcal{M}$  and, using condition (\*), find those admitting focal points. When possible, we show that  $\mathcal{M}$  is geodesically incomplete, thus singular. Our results have applications to the instability of extra space dimensions in string theory [2].*

## References

- [1] G. J. Galloway and J. M. M. Senovilla, “Singularity theorems based on trapped submanifolds of arbitrary co-dimension”, *Classical Quant. Grav.* **27** (2010), no. 15, 152002, 10 pp
- [2] R. Penrose, “On the instability of extra space dimensions” in *The future of the theoretical physics and cosmology*, G. W. Gibbons, E. P. S. Shellard, and S. J. Rankin, eds. (Cambridge, 2002), 185–201, Cambridge Univ. Press, Cambridge, 2003